V. S. Kirov, Yu. D. Kozhelupenko, and S. D. Tetel'baum

Approximate formulas are given which permit the determination of heat-transfer coefficients with sufficient accuracy for gases with Prandtl numbers from 0.3 to 1.

Mixtures of hydrogen and helium with "heavy" gases [1-3] and other gases can be used as working substances in power devices. In such mixtures, the Prandtl number may take on values from $\sim 1$ to $\sim 0.3$ depending on gas composition as shown in Fig. 1.

The curves given in the figure for the dependence of $\operatorname{Pr}$ on gas composition were constructed from theoretical values for viscosity and thermal conductivity determined by methods previously discussed [5]. For a mixture of helium and carbon dioxide, the values of the Prandtl number determined in this way agreed with experimentally determined values [4].

In the calculation of heat-transfer coefficient, it is necessary to have a computational formula for the determination of the heat-transfer surfaces and the selection of the optimal mixture composition which makes it possible to determine the heat-transfer coefficient during turbulent flow simply and reliably.

There is a whole set of relatively simple empirical relations for this purpose obtained from theoretical solutions and from experimental data (see Table 1). However, these formulas are suitable for gases having Prandtl numbers greater than $0.5-0.6$ or less than 0.05 . When using these formulas for gases with $\operatorname{Pr}<0.5$, the error in the determination of the heat-transfer coefficient may amount to $20-30 \%$ and more.

The integral of $R$. Lyon [6-8] yields the most precise solution; particularly simple solutions of this integral show a significant error in the region $\operatorname{Pr}=0.3-0.5$.

By analysis of theoretical solutions and existing experimental data of various authors on the heattransfer coefficient for completely turbulent flow of gases and liquids [7-10], it was established that Nu is directly proportional to $\operatorname{Pr}^{k}$, where the exponent $k$ is greater the smaller the Prandtl number. Thus $\mathrm{k}=0.8$ for liquid metals ( $\operatorname{Pr}<0.05$ ) and $\mathrm{k}=0.4-0.6$ for liquids and gases with $\operatorname{Pr} \geq 0.5$.


Fig. 1. Prandtl number of gas mixtures as a function of volume content of the "heavy" component: 1) hydro-gen-nitrogen mixture; 2) helium-carbon dioxide mixture; 3) hydrogen-carbon dioxide mixture.

[^0]TABLE 1

| Prandtl number, Pr | 0,3 |  |  |  | 0.5 |  |  | 1.0 |  | 'Reference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Reynolds number, Re | $10^{3}$ | $10^{\circ}$ | $10^{7}$ | $10^{5}$ | 10 | 107 | $10^{5}$ | 106 | $10^{7}$ |  |
| $\mathrm{Nu}=0.021 \mathrm{Re}^{0,8} \mathrm{P}^{0.43}$ | ' 124,5 $-22^{*}$ | 743.9 <br> -24 | 4630 -26 | $\begin{gathered} 156.0 \\ --10 \end{gathered}$ | $990$ | $\begin{array}{r} 6250 \\ -14 \end{array}$ | $\begin{array}{r} 210 \\ -\quad-5 \end{array}$ | $\begin{gathered} 1336 \\ -6 \end{gathered}$ | $\begin{array}{r} 8350 \\ -11 \end{array}$ | [6] |
| $\Lambda u=0,022 \mathrm{Re}^{0,8} \mathrm{Pr}^{0,0}$ | 109 --8 | 621 -7 | 4930 -15 | 145,2 <br> -2 | $\begin{array}{r} 915 \\ -6 \end{array}$ | $\begin{gathered} 5,70 \\ -50 \end{gathered}$ | $\begin{gathered} 220 \\ 0 \end{gathered}$ | $\begin{gathered} 1356 \\ -2 \end{gathered}$ | $\begin{gathered} 8750 \\ -7 \end{gathered}$ | [7] |
| $\mathrm{Nu}=\frac{0,023 \mathrm{Re}^{0,8} \mathrm{Pr}}{1-2,1+\mathrm{Re}^{-1}\left(\operatorname{Pr}^{-3}-1\right)}$ | 111 <br> -10 | 690 -8 | 3930 -88 | 153.5 | 90-4 | 5700 -1 | 230 -5 | 1450 -3 | 9110 -3 | [8] |

*The relative error in \% in comparison with Eq. (1) is given under the value of the Nusselt number.

Existing theoretical and experimental data on the heat-transfer coefficients of gases and liquids for Re in the range $10^{5}-10^{7}$ and for $\operatorname{Pr}>0.6$ and $\operatorname{Pr}<0.05[7,8]$ make it possible to obtain an approximate formula for $\operatorname{Pr}$ in the range $0.3-1$ with the help of the Lyon integral. Furthermore, the structure of the expression for Nu typical of gases and liquids with $\operatorname{Pr}>0.6$ and $\operatorname{Re}>10^{5}$ is preserved, but the exponent of Pr in this formula is a function of the absolute value of the Prandtl number:

$$
\begin{equation*}
\mathrm{Nu}=0.022 \mathrm{Re}^{\mathrm{r} .8} \mathrm{Pr}^{k} . \tag{1}
\end{equation*}
$$

where $\mathrm{k}=0.595 \mathrm{Pr}^{-0.126}$.
This expression agrees within $1 \%$ with the solution obtained from the Lyon integral for the corresponding conditions and can be simplified and brought to the form

$$
\begin{equation*}
\mathrm{Nu}=0.022 \mathrm{Re}^{0.8} \mathrm{Pr}^{0.06} \tag{2}
\end{equation*}
$$

with an increase in the error to $5 \%$.
An approximation to the exact solution of Eq. (1) is possible with an accuracy of no worse than $10-15 \%$ in the form typical of liquid metals in the range of $\operatorname{Pr}$ and Re under discussion.

Table 1 gives a comparison of the various computational formulas and their relative errors in comparison with the exact theoretical solution obtained from the Lyon integral [8]. One should note here that the exact solution of the Lyon integral, just like Eq. (1), was compared for certain values of $\operatorname{Pr} \leq 0.05$ and $\operatorname{Pr}>0.6$ with experimental data for number of liquids [9, 10]. The comparison indicated extremely good agreement.

Therefore for comparison of the expressions in the table, we took for exact ("standard") values of Nu those values of this criterion which were obtained from a solution of the Lyon integral.

As is clear from Table 1, the proposed approximate formulas yield the greatest accuracy, which makes it possible torecommend them for the calculation of heat-transfer coefficients of gas mixtures containing He and $\mathrm{CO}_{2}$, and $\mathrm{H}_{2}$ and $\mathrm{CO}_{2}$. However, they need experimental verification.

## NOTATION

Nu is the Nusselt number;
Re is the Reynolds number;
$\operatorname{Pr}$ is the Prandtl number.

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